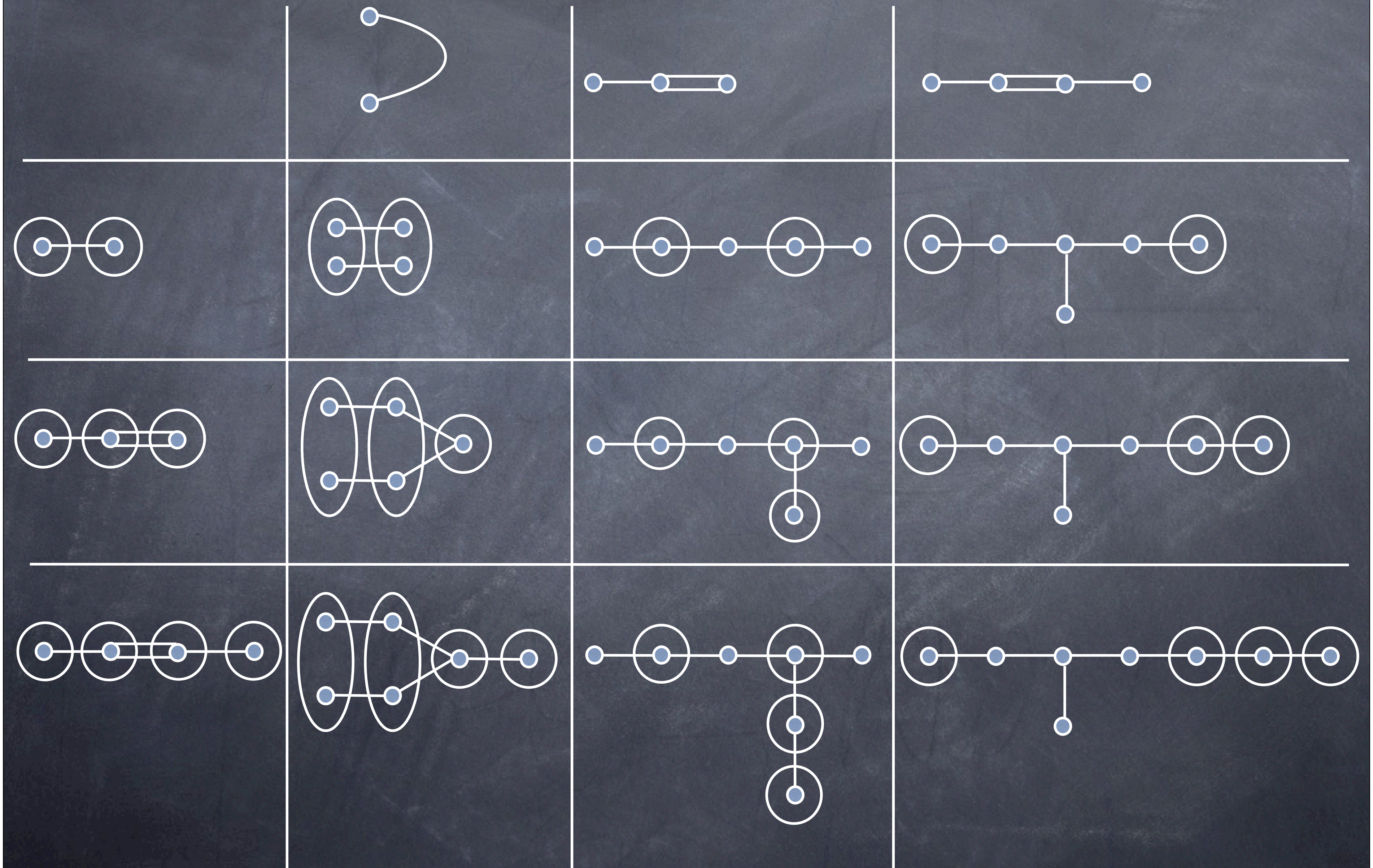
A Geometric introduction to the Freudenthal-Tits Magic Square

HVM

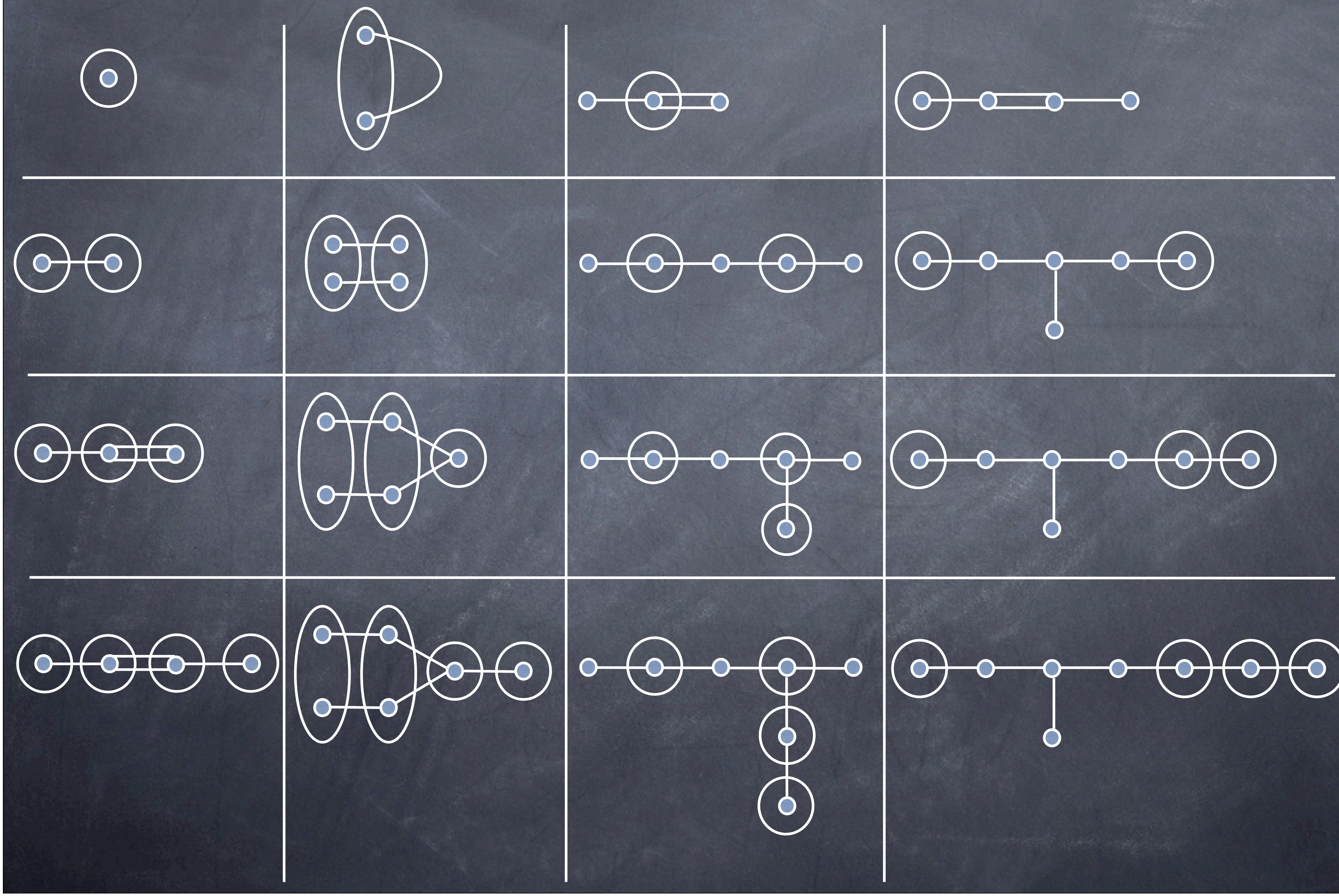
Gent, October 19, 2012

`DAG'-Day 2012

Freudenthal-Tits Magic Square

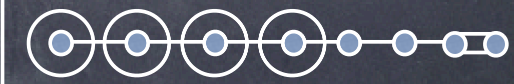


Freudenthal-Tits Magic Square



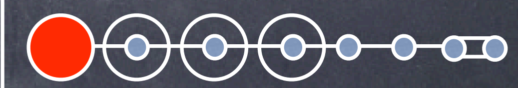
The Magic Game

Start with self-opposite Galois involution



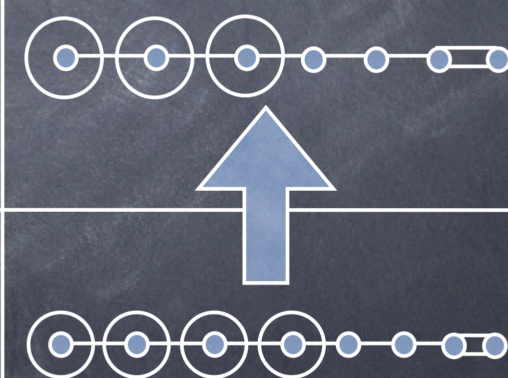
The Magic Game

Take residue in fixed vertex



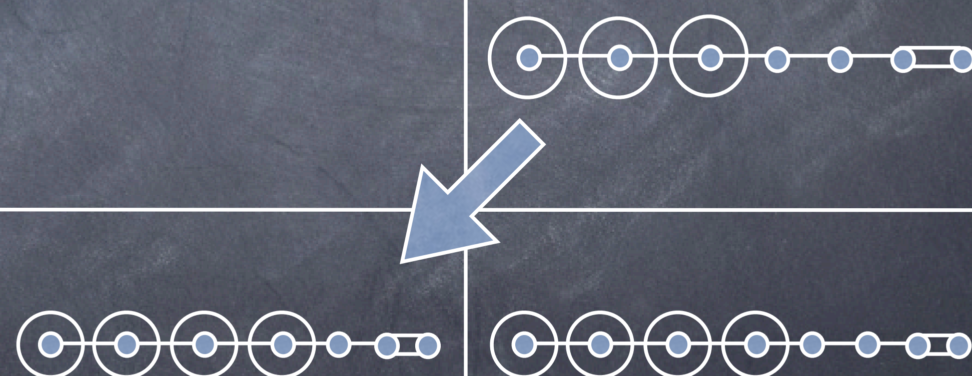
The Magic Game

Get Galois involution in residue



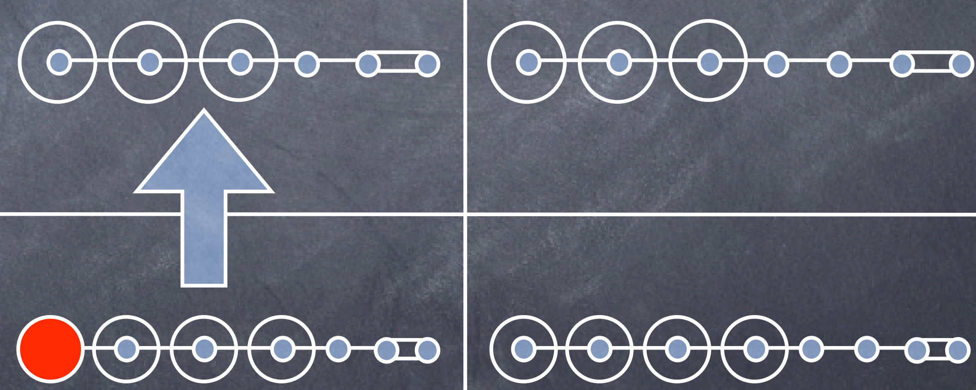
The Magic Game

Take opposite of this Galois involution



The Magic Game

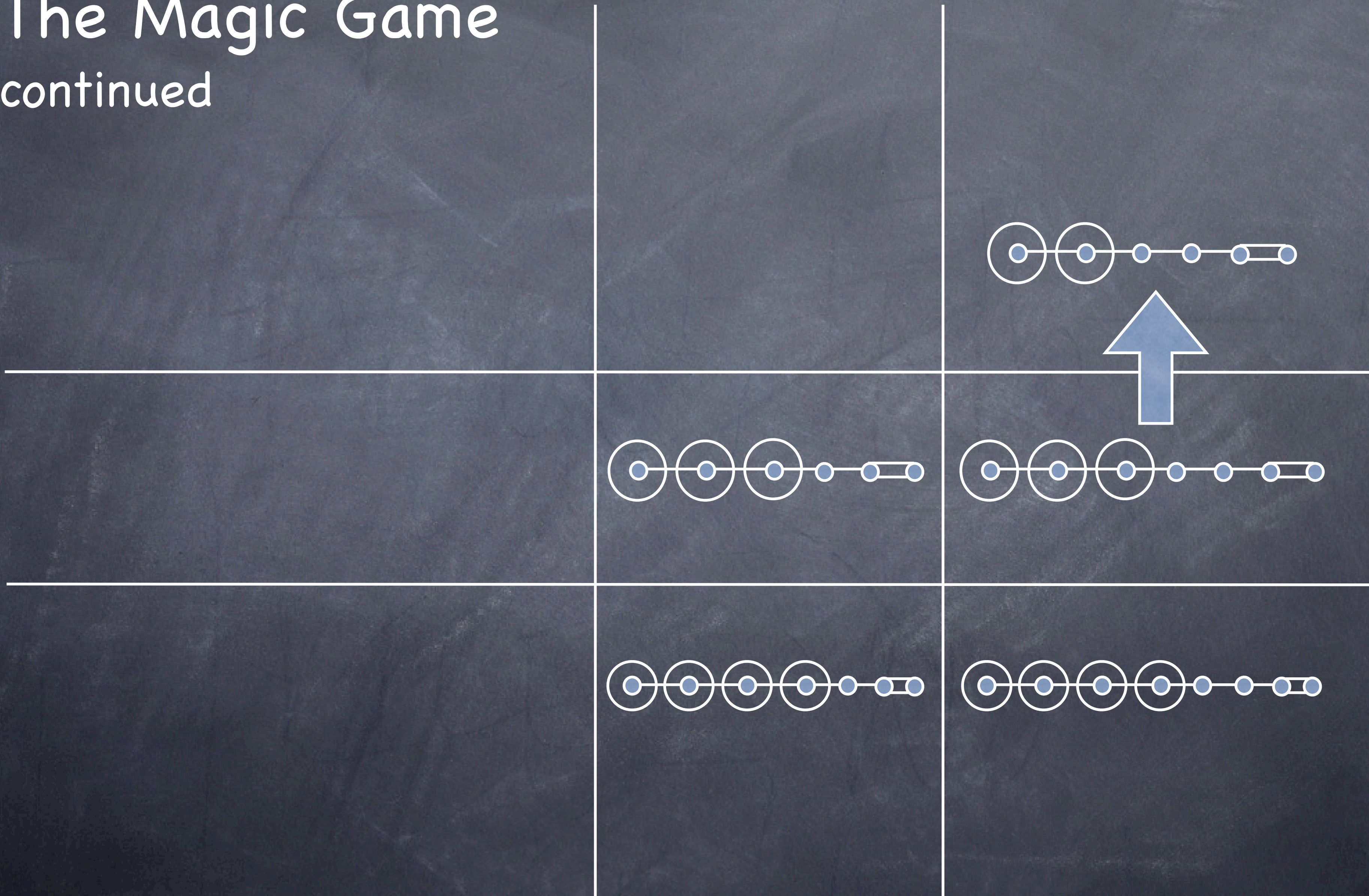
Then there exists a residue of a fixed vertex such that the induced involution is a self-opposite Galois involution



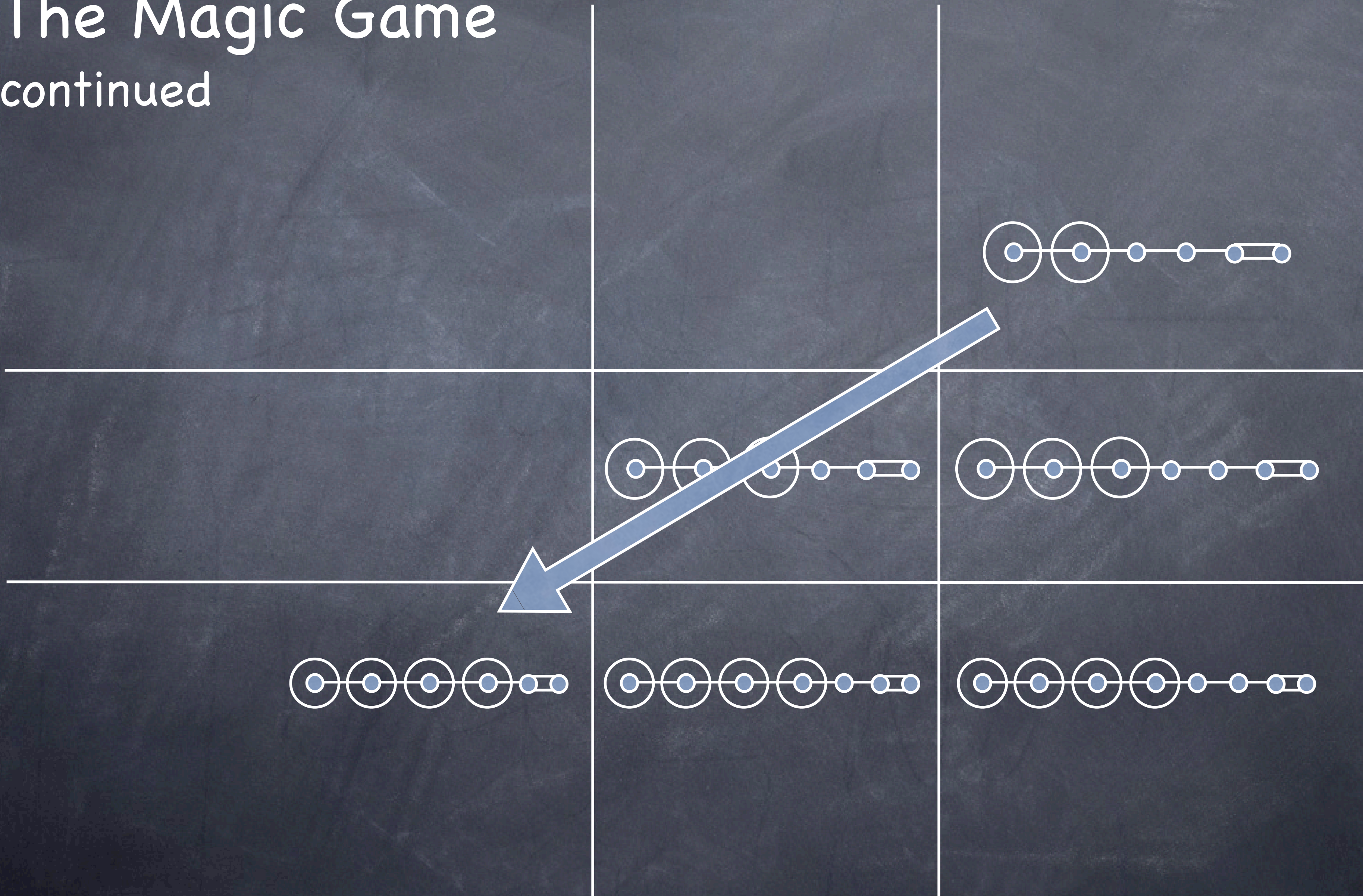
The Magic Game

continued

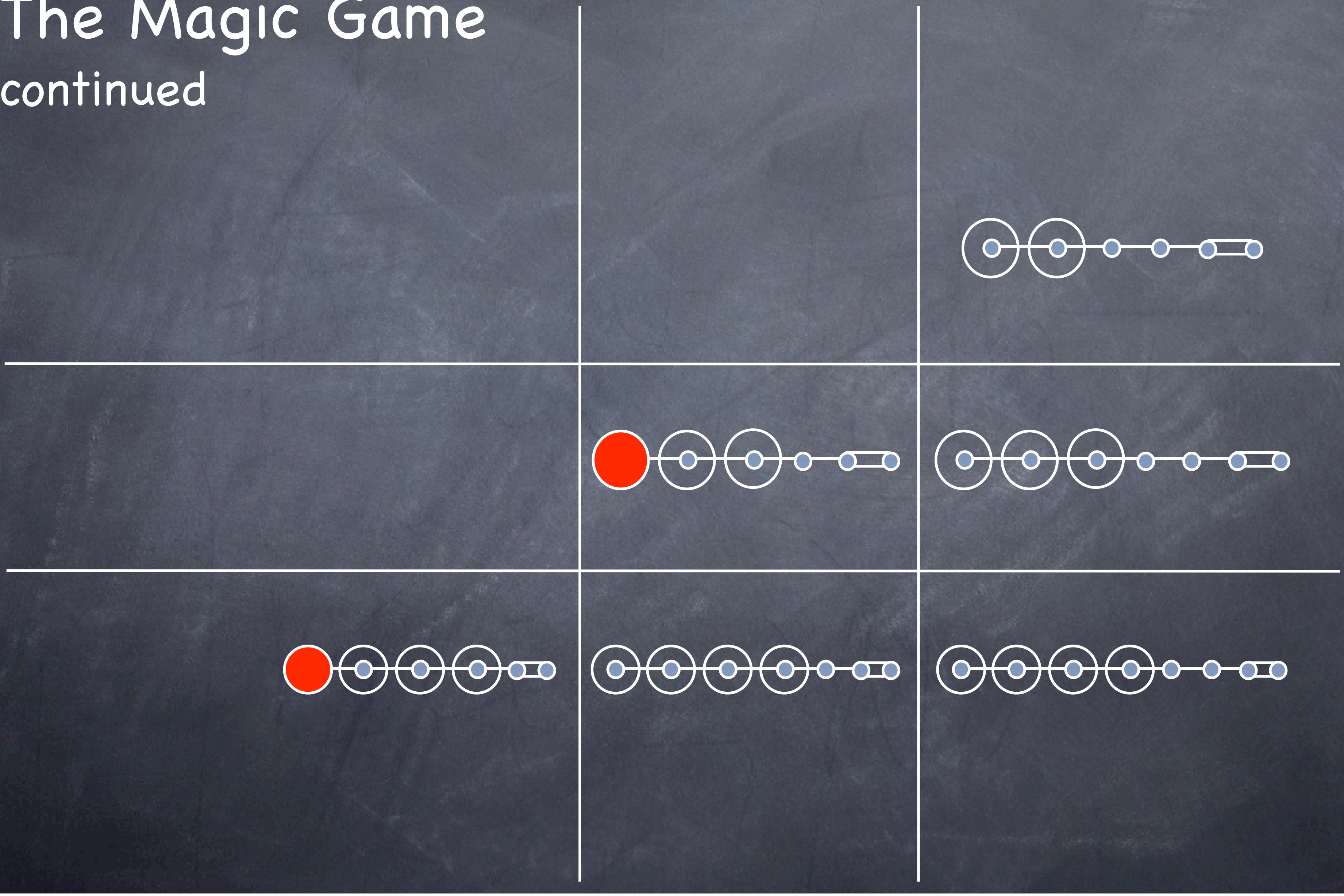
The Magic Game
continued



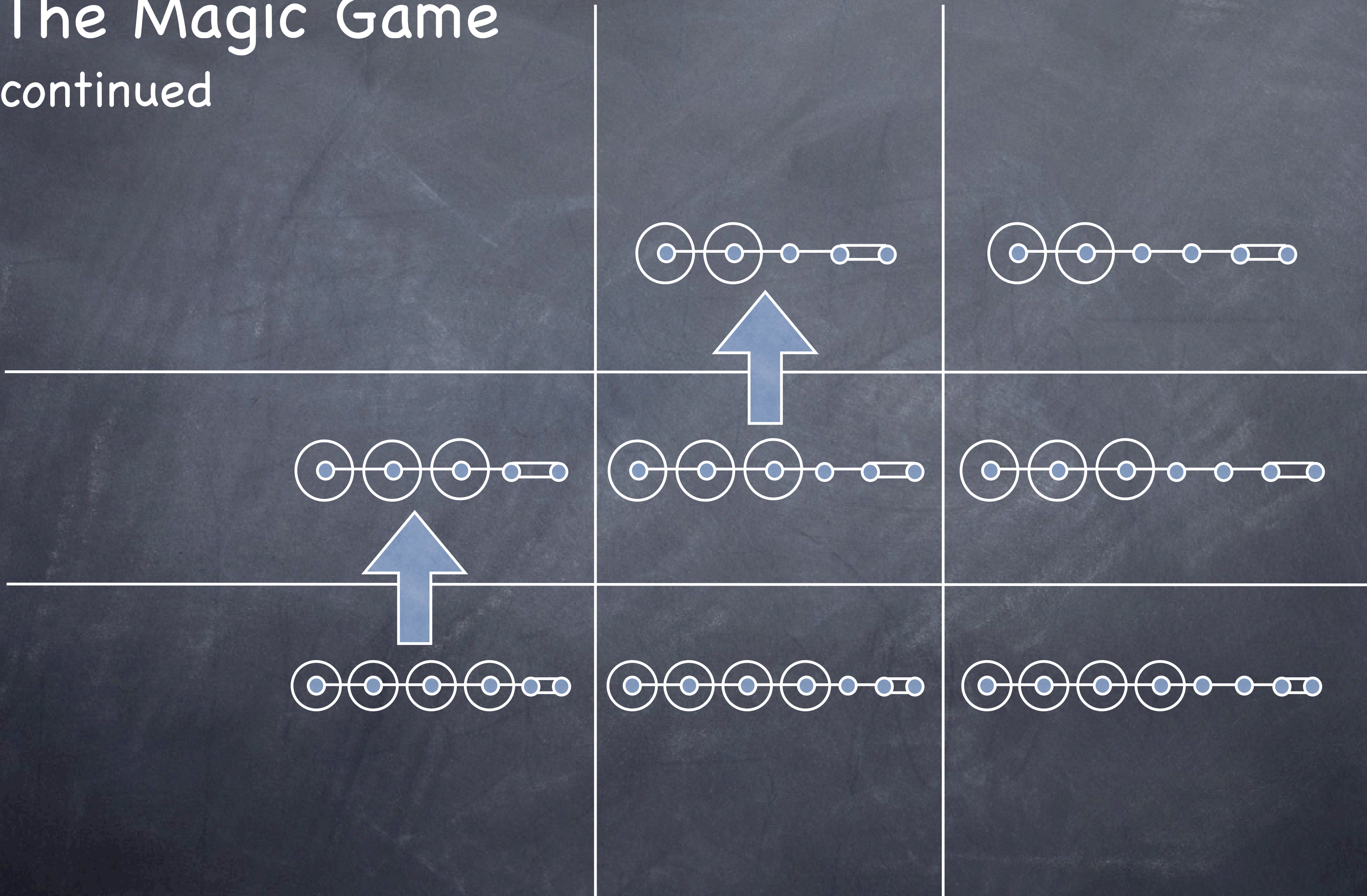
The Magic Game
continued



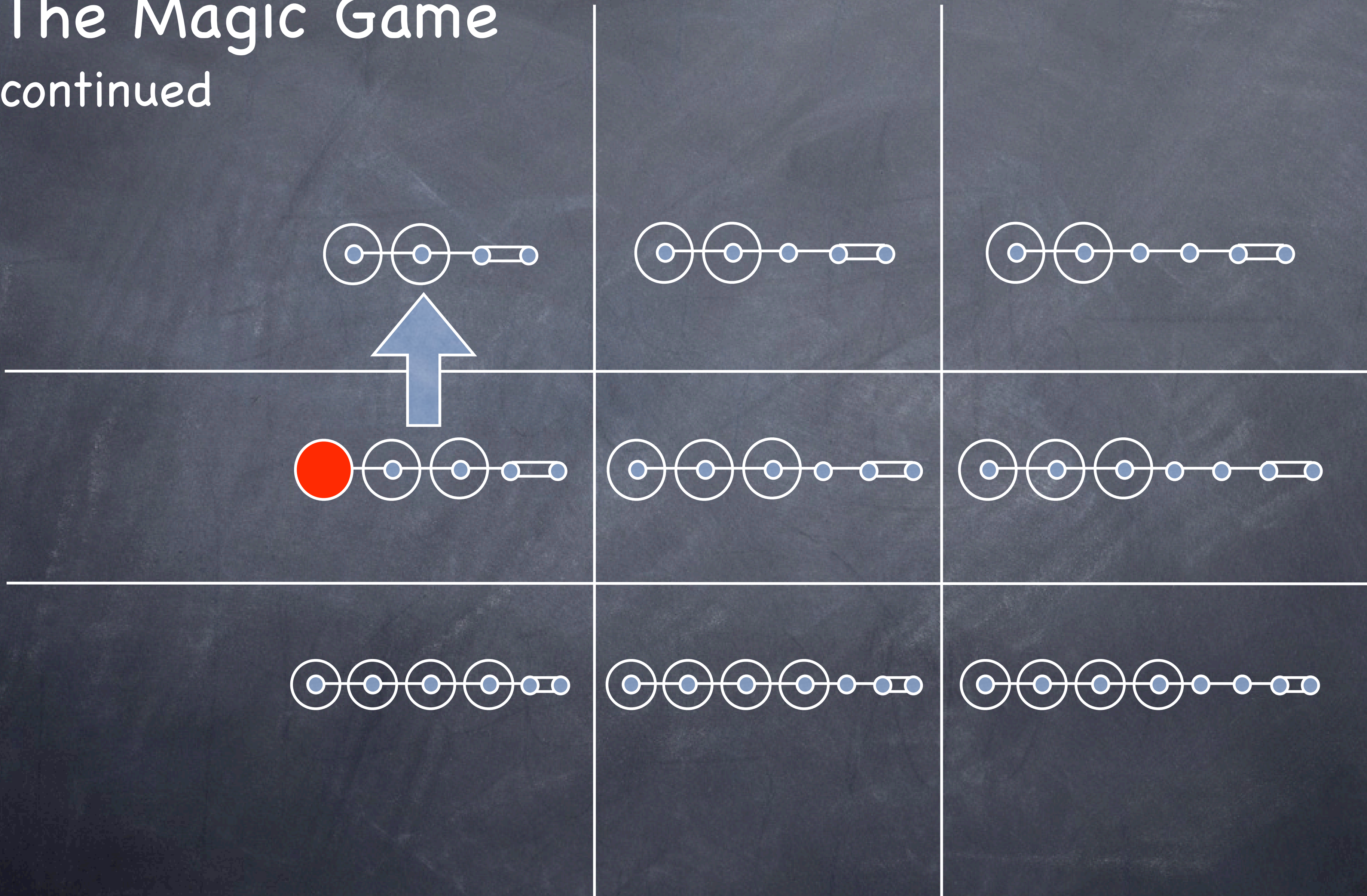
The Magic Game
continued



The Magic Game
continued

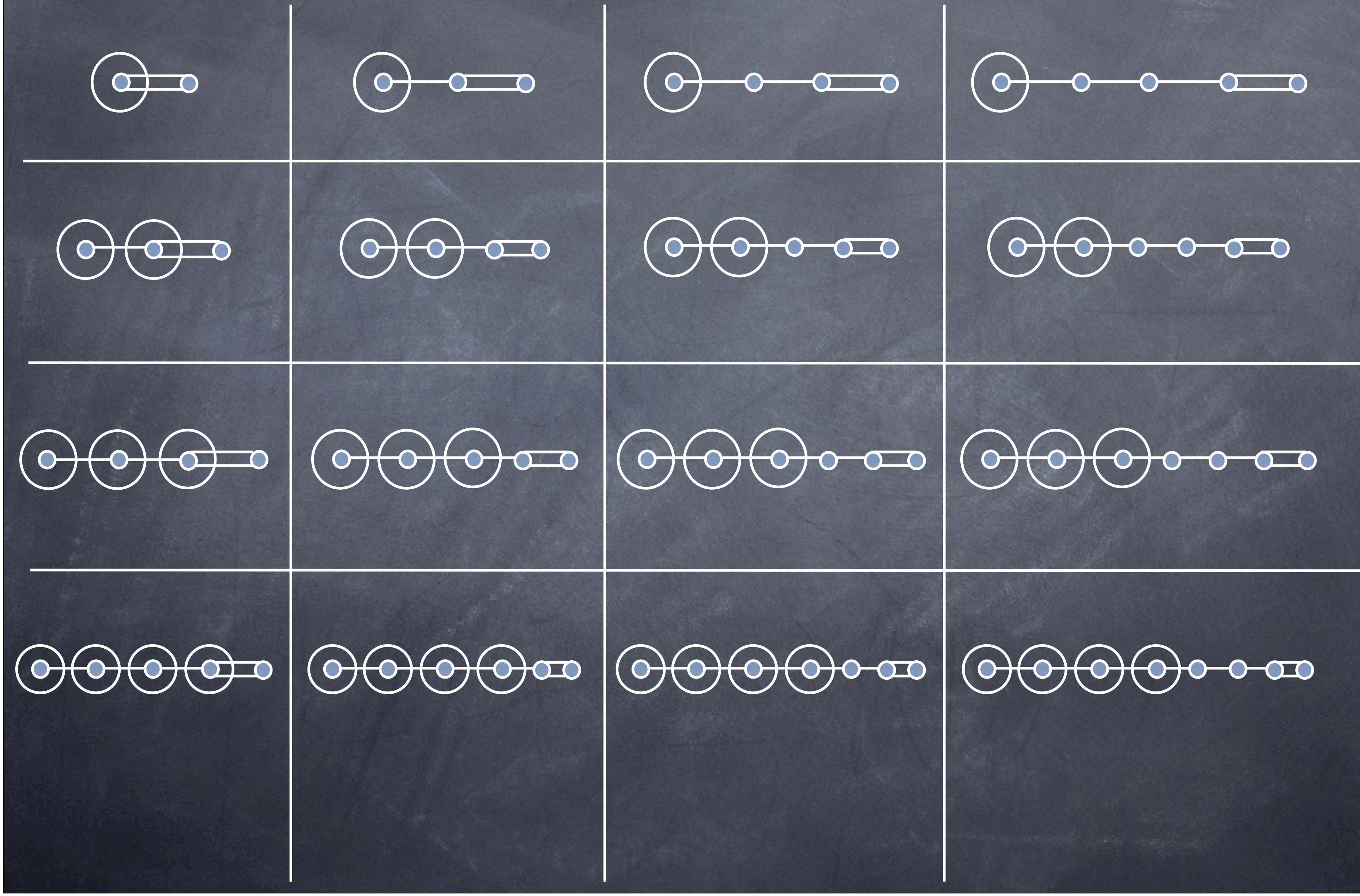


The Magic Game
continued



The Magic Game
continued

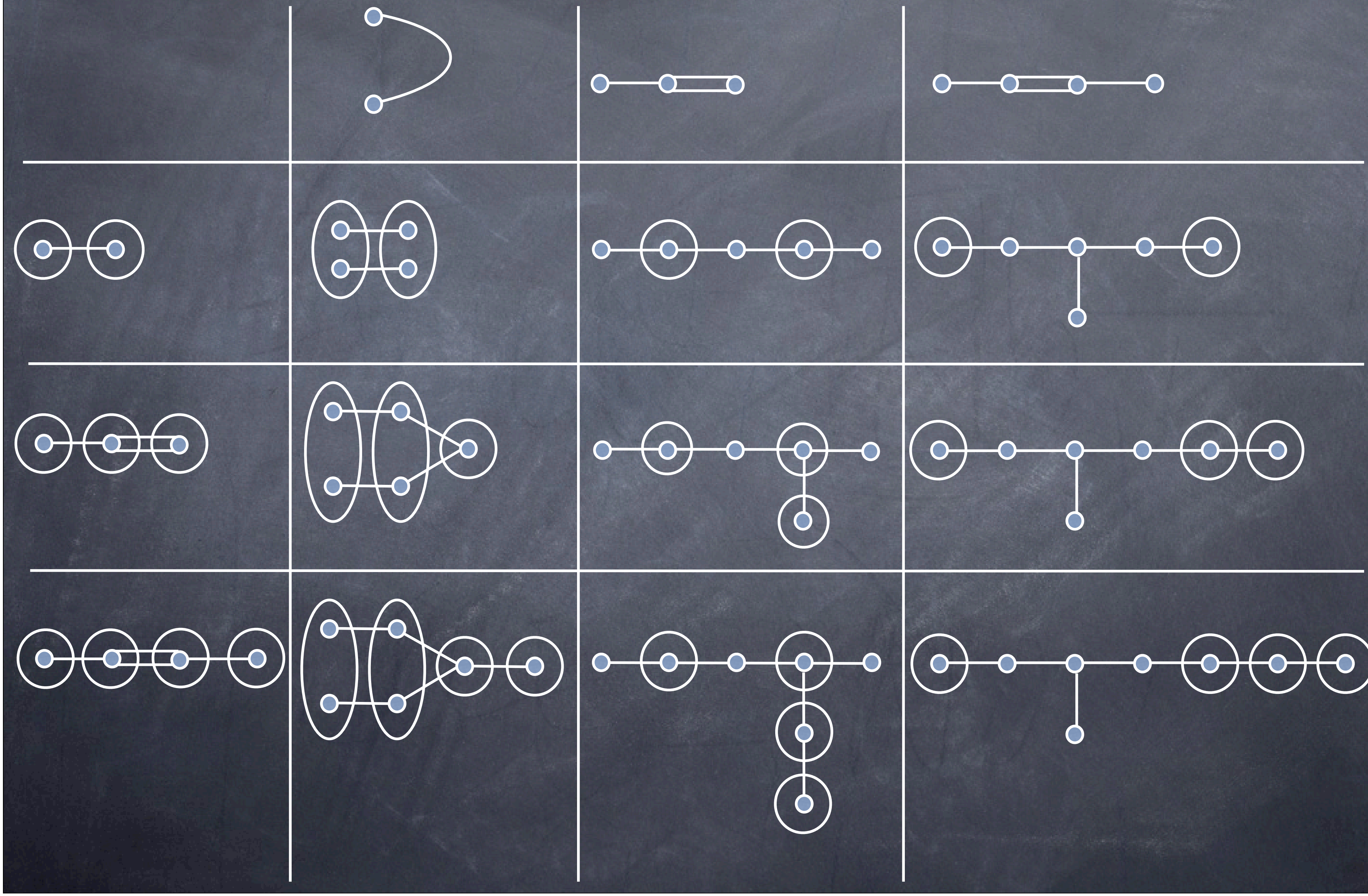
B_8 Square



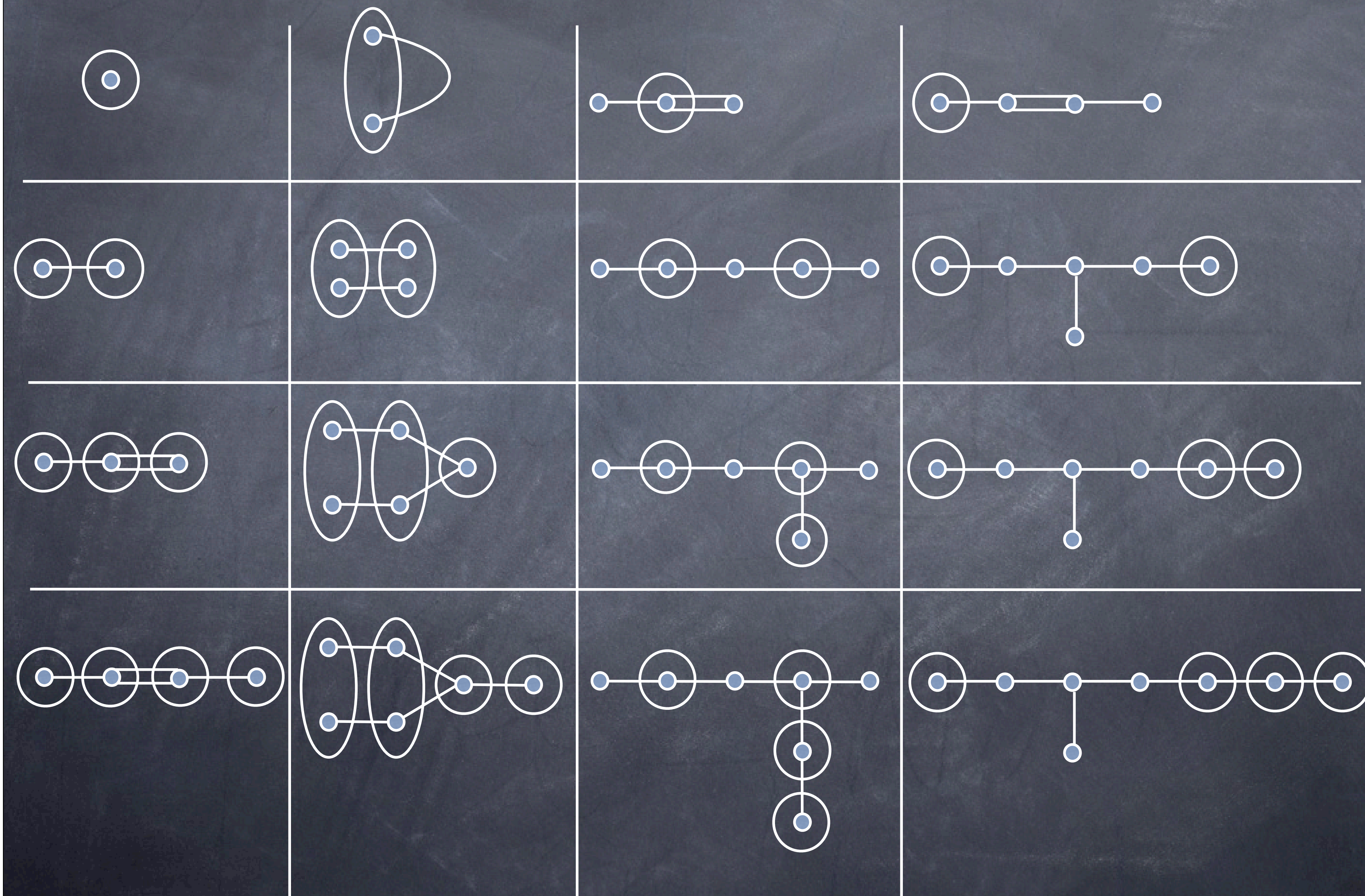
The Freudenthal-Tits Magic Square

* Start with E8 example

Freudenthal-Tits Magic Square



Freudenthal-Tits Magic Square



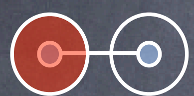
The second row of the Square

The second row of the Square

The second row of the Square

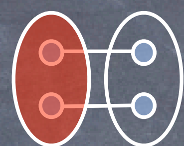
The second row of the Square

A_2



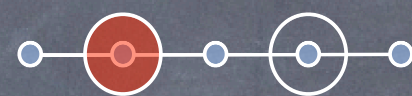
Projective plane

$A_2 \times A_2$



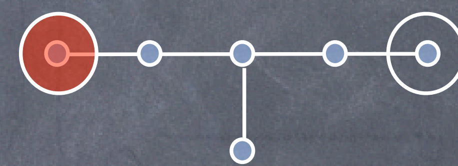
Tensor product of 2 projective planes

A_5



Line Grassmannian of projective 5-space

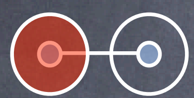
E_6



The $E_{6,1}$ parapolar space

The second row of the Square

A_2

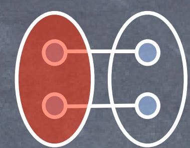


Projective plane

Veronese Variety

V_2

$A_2 \times A_2$

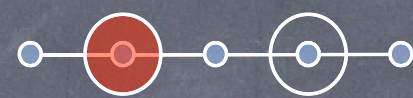


Tensor product of 2 projective planes

Segre Variety

$S(2,2)$

A_5

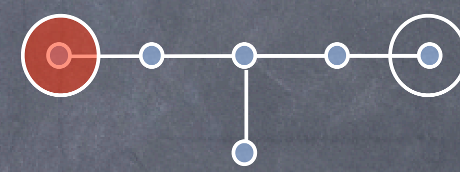


Line Grassmannian of projective 5-space

Grassmann Variety

$G(2,6)$

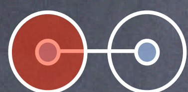
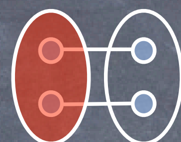
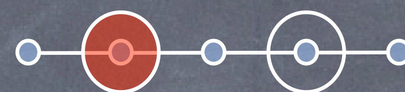
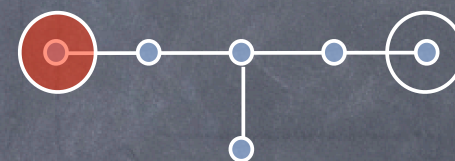
E_6



The $E_{6,1}$ parapolar space

26-dim $E_{6,1}$ Variety

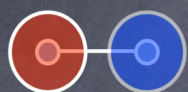
The second row of the Square

A_2	$A_2 \times A_2$	A_5	E_6
			
Projective plane	Tensor product of 2 projective planes	Line Grassmannian of projective 5-space	The $E_{6,1}$ parapolar space
Veronese Variety	Segre Variety	Grassmann Variety	26-dim $E_{6,1}$ Variety
V_2	$S(2,2)$	$G(2,6)$	

S E V E R I   V A R I E T I E S

The second row of the Square

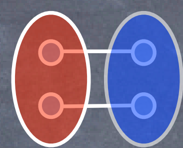
A_2



Projective plane

Veronese Variety

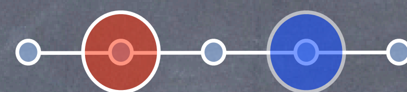
$A_2 \times A_2$



Tensor product of 2 projective planes

Segre Variety

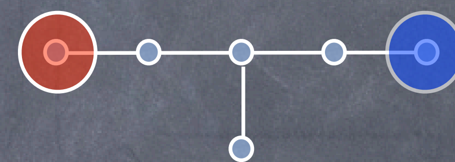
A_5



Line Grassmannian of projective 5-space

Grassmann Variety

E_6



The $E_{6,1}$ parapolar space

26-dim $E_{6,1}$ Variety

Projective planes over (quadratic, not necessarily associative) algebras with zero divisors

Veronese varieties

Mazzocca-Melone axioms

X a set of points of PG(V), V finite vector space
Y a set of planes of PG(V) such that $y \cap X$ is a conic, $\forall y \in Y$

(MM1) Two points are contained in a member of Y
(MM2) Two members of Y intersect in subset of X
(MM3) The tangent lines at $x \in X$ to all conics through x are contained in a 2-space

Let's generalize

Mazzocca-Melone axioms, generalized

X a set of points of PG(V), V ~~finite~~ any vector space

Y a set of ~~planes~~ (n+1)-spaces of PG(V) such that $y \cap X$ is a ~~conic~~ n-dimensional quadric, $\forall y \in Y$

(MM1) Two points are contained in a member of Y

(MM2) Two members of Y intersect in subset of X

(MM3) The tangent ~~lines~~ spaces at $x \in X$ to all ~~conics~~ quadrics through x are contained in a ~~2~~ 2n-space

Take all quadrics with maximal Witt index

Then we have the following theorem (joint with Jeroen Schillewaert):

Every Mazzocca-Melone set with |Y|>1 is one of:

(i) Quadric Veronesean variety V_2
(ii) Segre variety S(1,2), S(1,3) or S(2,2)
(iii) Grassmann variety G(2,5) or G(2,6)
(iv) Half-spin variety $D_{5,5}$
(v) 26-dimensional $E_{6,1}$ variety

Corollary

If all quadrics are n-dimensional and with maximal Witt index, and if dim(V)≥3n+2,
then dim(V)=3n+2 and every Mazzocca-Melone set is a Severi variety, more exactly, one of:

(i) Quadric Veronesean variety V_2, n=1
(ii) Segre variety S(2,2), n=2
(iii) Grassmann variety G(2,6), n=4
(iv) 26-dimensional $E_{6,1}$ variety, n=8

Thank you